Attenuation of Sunlight by Saturn's Rings

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Short title: Saturn Ring Shadowing

Version: 2 November 2005

Submitted:

Revised:

Accepted:

Abstract

The pattern of shadows cast upon Saturn by its ring system, and their effect on sunlight reaching the atmosphere of Saturn, is derived as a function of latitude, local time, and season. During the Cassini era (early 2005, near southern summer) sunlight is attenuated by up to two orders of magnitude across a 60^{o} range in latitude. For the Voyager 2 fly-by (26 August 1981, near Saturn equinox) the attenuation is highly concentrated, and can reach a maximum of 19 orders of magnitude, although only a few degrees of latitude are shadowed.

1. Introduction

Saturn's unique system of rings attenuates sunlight and therefore alters the insolation pattern at the top of Saturn's atmosphere. The pattern of shadows cast upon Saturn by its rings varies according to season; the widest range of shadowed latitudes occurs during solstice. Furthermore, a point in Saturn's shadowed hemisphere will be attenuated by different portions of the rings over the course of a day. Therefore, correct determination of the variation of insolation at Saturn as a function of latitude, local time, and season is necessary for accurate aeronomical calculations.

Brinkman and McGregor [1979] provided the first quantitative study of the effect of Saturn's rings on solar radiation incident on the top of the atmosphere, using data from *Allen* [1963] and *Cook et al.* [1973] to define the position and optical depth of the ring system. Errors in the equations of *Brinkman and McGregor* [1979] led to an independent derivation by B. Baker, S. Postawko, W. Kuhn, H. Waite, and S. Atreya, although only the results from this derivation were presented [see *Waite*, 1981]. Soon after, *Van Hemelrijck* [1986] updated the study of *Brinkman and McGregor* [1979] by correcting the errors in their expressions, and by using an improved model of Saturn's rings based on Voyager 2 measurements [*Esposito et al.*, 1983]. *Van Hemelrijck* [1986] calculated mean diurnal and annual insolation, and demonstrated that shadowing by Saturn's rings can reduce mean insolation in a winter hemisphere by as much as 50%.

This paper presents a new derivation of the effects of Saturn's rings on insolation at the top of the atmosphere, with a focus on diagnosing the instantaneous attenuation as a function of latitude, local time, and season. The hope is that this derivation will prove useful and accessible for other researchers in the field, and towards that end a computer program that performs the calculations described here can be retrieved from the Boston University Aeronomy website (http://sirius.bu.edu/planetary/saturn.html).

2. Observations

The optical depth of Saturn's rings has been measured at radio [*Marouf et al.*, 1986], ultraviolet [*Esposito et al.*, 1983] and infrared wavelengths [*Nicholson et al.*, 2000]. The remarkable constancy of ring optical depth observed in both wavelength and time (with 9 years separating the measurements described above) indicates that the majority of forward attenuation in Saturn's rings can be attributed to solid body scattering, a description that fits well with particle size distributions in the rings, ~1 cm – 20 m [*Marouf et al.*, 1986; *French and Nicholson*, 2000]

In addition to solid body scattering, absorption by a ring atmosphere (or other gases in the Saturn system) must be considered. The rings are comprised mostly of water ice, and so water products will all be present in various quantities. *Broadfoot et al.* [1981] found an H atmosphere with number density ~600 cm^{-3}, and column density ~10^{13} cm^{-2}, but no other constituents with

UV signatures. *Doyle et al.* [1996] found a comparable cloud of OH, with $n \sim 700$ cm^{-3} and $N_{OH} \sim 10^{13}$ cm^{-2}. Measurements of the Titan hydrogen torus show that it is less dense by an order of magnitude [*Broadfoot et al.,* 1981]. Recent measurements by the UVIS instrument aboard Cassini show a variable oxygen emission concentrated near Saturn's rings with a total of 10g of oxygen observed within a 10 R_S radius [*Esposito et al.*, 2005]. Depending on the wavelengths of light one is interested in, the various gases in the Saturnian system may or may not play an important role in attenuating sunlight. In the calculations described below, only the effect of the rings of Saturn shadowing its atmosphere will be considered.

3. Model

When studying solar-driven processes in Saturn's atmosphere, a question of primary importance is: What is the solar irradiance for a specific position at a specific time on Saturn? This question is answered partially by relatively simple solar zenith angle effects, but is complicated by the attenuation of sunlight by Saturn's rings, which varies strongly with season, time, and position on Saturn. Thus, the goal here is to derive $I(\theta,\delta,\phi)$, which describes the solar flux incident at the top of Saturn's atmosphere for a given local time (θ), latitude (ϕ) and solar declination (δ). The derivation will progress in parts by building upon equalities illustrated in the geometry of Figs. 1 and 2. A brief example calculation follows this derivation and is described below in Section 4.1.

First, from Fig. 1 it is clear that r_R can be determined by summing the sides of the right triangle.

$$\left(r_R + r_S\right)^2 + \left(R_S \sin\theta\right)^2 = \left(R_R + R_S\right)^2 \tag{1}$$

and of course r_S is itself a function of θ,

$$r_S = R_S \cos\theta \tag{2}$$

Here R_S is the radius of Saturn (or to a specific altitude above the disk of Saturn), and R_R is the radius of the rings that attenuates sunlight at the wavelength(s) considered. Equation (1) is quadratic in r_R,

$$r_R^{\,2} + r_R\left(2R_S\cos\theta\right) - \left(2R_S R_R + R_R^{\,2}\right) = 0 \tag{3}$$

and therefore,

$$r_R = -R_S\cos\theta + \sqrt{R_S^{\,2}\cos^2\theta + 2R_S R_R + R_R^{\,2}} \tag{4}$$

The variable r_R describes the length of the segment of the rings that will cast a shadow on a meridional slice of Saturn. Note that this two-dimensional slice can be drawn as a circle (Fig. 2) with radius r_S, where the length of r_S is given in Eq. (2). Next, in order to determine the length r, three equalities involving the sides and angles indicated in Fig. 2 can be constructed. As can be seen from the figures, r describes the radial position on the rings that lies in a direct line between the Sun and a point at Saturn ($r < r_R$).

$$\cos\phi = \frac{\left(r_S - a\right)}{r_S} = 1 - \frac{a}{r_S}, \text{ which can be rewritten as}$$

$$a = r_S\left(1 - \cos\phi\right) \tag{5}$$

$$c^2 + \left(r_S - a\right)^2 = r_S^{\,2} \tag{6}$$

$$c = b + a\tan\delta = \left(r + a\right)\tan\delta \tag{7}$$

A substitution of c from Eq. (7) into Eq. (6), results in a quadratic in r,

$$r^2\left(\tan^2\delta\right) + r\left(2a\tan^2\delta\right) + \left(a^2\sec^2\delta - 2ar_S\right) = 0 \tag{8}$$

and therefore,

$$r = -a\left\{1-\sqrt{\cot^2\delta\left(2\frac{r_S}{a}-1\right)}\right\} \tag{9}$$

where a and r_S are given in Eqs. (5) and (2) respectively. From Fig. 1 it is clear that $r_R \geq R_R$, and also that $r \geq r_*$, so a correction for local time must be made in order to accurately determine the optical depth of the rings that is responsible for shadowing.

$$r_* = \frac{r}{r_R}R_R \tag{10}$$

Finally, the attenuation of sunlight can be calculated by applying a known distribution of optical depth along the rings. The optical depth of Saturn's rings is typically expressed normal to the ring plane, so those measurements can be extrapolated to other insolation angles by

$$I = I_0 \exp\left\{\frac{-\tau\left(r_*\right)}{\sin\delta}\right\} \tag{11}$$

where τ is the normal optical depth, and I and I_o are the emergent and incident flux, respectively.

These calculations have considered a spherical planet. In order to determine the attenuation of sunlight for an oblate Saturn, Eq. (2) needs to be altered, and can be approximated as

$$r_S \simeq R_S\left(1-e\cos^2\theta\right)\cos\theta \tag{12}$$

where R_S refers to the equatorial radius of Saturn here, and e the oblateness. The effects of considering an oblate Saturn rather than a spherical Saturn are minimal for ring shadowing calculations, as there is relatively little solar insolation at high latitudes where the oblateness is more dramatic.

4. Results

In order to demonstrate the effects of Saturn's rings shadowing its atmosphere, a model of the optical depth as a function of ring radius was adapted from the *Esposito et al.* [1983] Voyager 2 photopolarimeter system (PPS) results. Figure 3 gives the ring model as optical depth normal to the ring plane as a function of Saturn radius. In this description, the ring radius from Fig. 1, R_R, is 1.4 R_S (spanning 1 – 2.4 Saturn radii). The ring model was chosen based on its relative simplicity, therefore making it easy to implement and modify. Calculations using a ring optical depth model with the finer radial structure displayed in the data, however, do not differ dramatically from those using the simplified model, as the shadowed latitudes pass quickly beneath each portion of the rings during the course of a Saturn day.

The attenuation of sunlight by Saturn's rings as a function of local time, latitude, and season, as expressed by Eq. (11), is given in Fig. 4. The top panel shows I/I_o values calculated for the Cassini era (i.e., near southern summer, $\delta = -24^o$), while the bottom panel plots I/I_o values for the Voyager 2 fly-by (26 August 1981, near equinox, $\delta = 2.7^o$). Ring shadowing is much more dramatic when Saturn is near equinox (maximum attenuation of 19 orders of magnitude for Fig. 4b), although the range of affected latitudes is also reduced during these times. Such a dark shadow can lead to troughs of reduced ionospheric electron density, and hence large meridional gradients in the ionosphere [*Mendillo et al*, 2005].

4.1 Sample Calculation

The ring shadowing derivation described above is elucidated here by a sample calculation that answers the question: what portion of the rings shadows 30^o N latitude during southern summer at 15 LT? This example will consider a spherical Saturn.

First, using the ring model adapted from the *Esposito et al.* [1983] results, $R_S \simeq 1.07$ R$_S$, and $R_R = 1.27$ R$_S$. (The variable R_S is greater than 1 R$_S$ because the shadowing calculation is made for the top of Saturn's atmosphere, taken as 4000 km here.) In other words, above 2.27 R$_S$ the optical depth of the rings is zero, and between 1.07 - 2.27 R$_S$ the normal optical depth of the rings is given by Fig. 3. The solar declination during southern summer is -26.73^o. At 15 LT (i.e., $\theta = 45^o$), $r_S \simeq 0.757$ R$_S$ from Eq. (2), and therefore $r_R \simeq 1.46$ R$_S$ from Eq. (4). Next, the value r is calculated from Eq. (9), giving $r \simeq 0.650$ R$_S$. (Note that the length of a for 30^o N latitude is ~ 0.101 R$_S$.) During local times other than noon, $r > r_R$, so a correction is made via Eq. (10), giving the new value, $r_* \simeq 0.566$ R$_S$. The value r_* is a length along the ring plane, and so is added to R_S in order to find the radius as given in Fig. 3. As 1.566 R$_S$ just catches the beginning of the B ring, sunlight passing through the rings under these conditions and hitting the top of the atmosphere at 30^o N latitude passes through a normal optical depth of 0.83 (see Fig. 3). The final attenuation is estimated from Eq. (11), which gives $I \simeq 0.158\ I_o$. By repeating this process for all latitudes and local times, it is possible to construct an attenuation map as in Fig. 4.

5. Summary

A derivation of the attenuation of sunlight by Saturn's rings as a function of solar declination, latitude, and local time has been presented. The focus has been to present a methodology that can easily be adapted and applied by other researchers. Attenuation profiles, and their effects on the ionosphere and plasmasphere of Saturn have been calculated using this method in *Moore et al.* [2004], *Mendillo et al.* [2005], and *Moore and Mendillo* [2005]. A version of the computer routine used in those studies, is available at the Boston University Aeronomy website (http://sirius.bu.edu/planetary/saturn.html). Clearly, correct determination of the pattern of shadows cast by Saturn's rings on the top of its atmosphere is important for any solar-driven processes within the atmosphere.

Acknowledgements. Thanks are due to Dr. M. Mendillo for helpful comments and a careful reading of the manuscript. Without his guidance, and also the encouragement of Dr. J. Moses, this work would not have been completed. Support comes from the NASA Graduate Student Research Program.

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Figure Captions

Figure 1:

Schematic of view looking down at Saturn and its rings from above, demonstrating how the radius of the circle to be shadowed, r_S, changes with increasing local time, θ. See text for descriptions of the various variables depicted. Not to scale.

Figure 2:

Schematic of the shadowed N-S slice of Saturn at local time θ. Variables are identified that allow calculation of the radius of Saturn's rings that lies in a line between Saturn and the Sun. The point being shadowed by the rings is indicated by a light gray dot. Not to scale.

Figure 3:

Ring optical depth model adopted from the Voyager PPS stellar occultation measurements [*Esposito et al.*, 1983]. Note that the values depicted here are for normal incidence to the ring plane.

Figure 4:

Calculation of the attenuation of sunlight for two seasons at Saturn: (a) southern summer during the Cassini era (δ = -24°), and (b) near equinox during the Voyager 2 fly-by (δ = 2.7°). The attenuation is expressed as the logarithm of Eq. (11), I/I_o. Note that the scales for each panel are different. Panel (a) plots a maximum attenuation of $10^{-1.9}$ (i.e., $I/Io \approx 0.013$) over a range of 90° in latitude; panel (b) plots a maximum attenuation of 10^{-19} over 30° of latitude

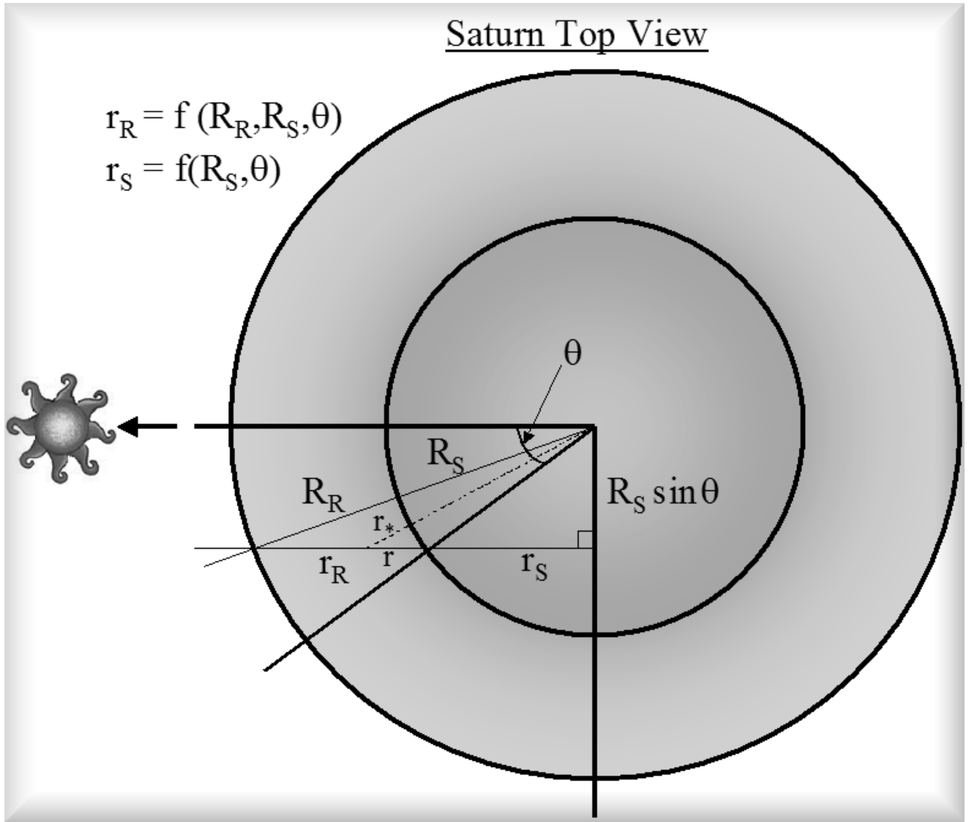

Figure 1

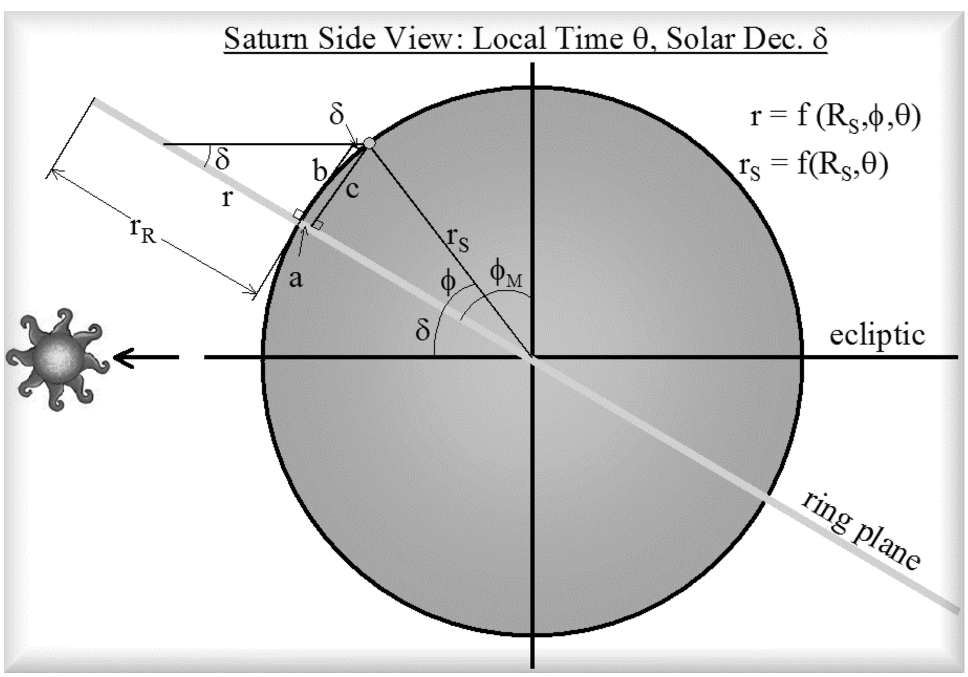

Figure 2

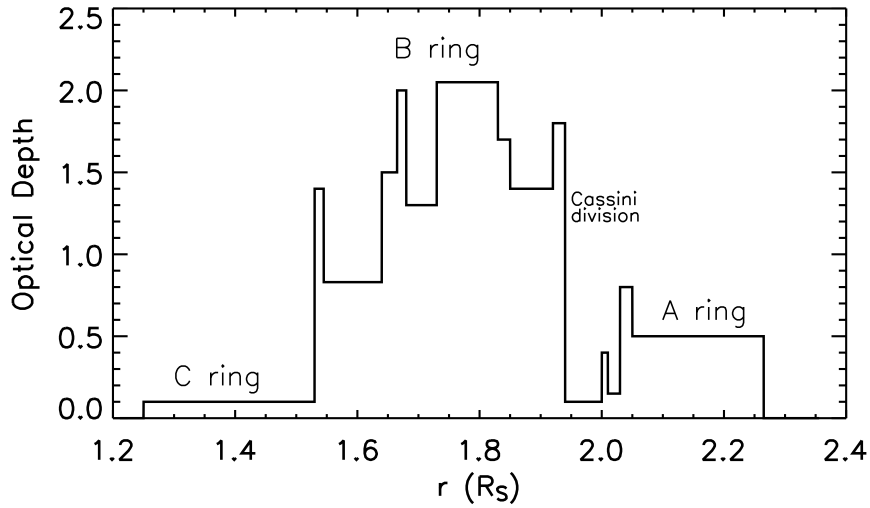

Figure 3

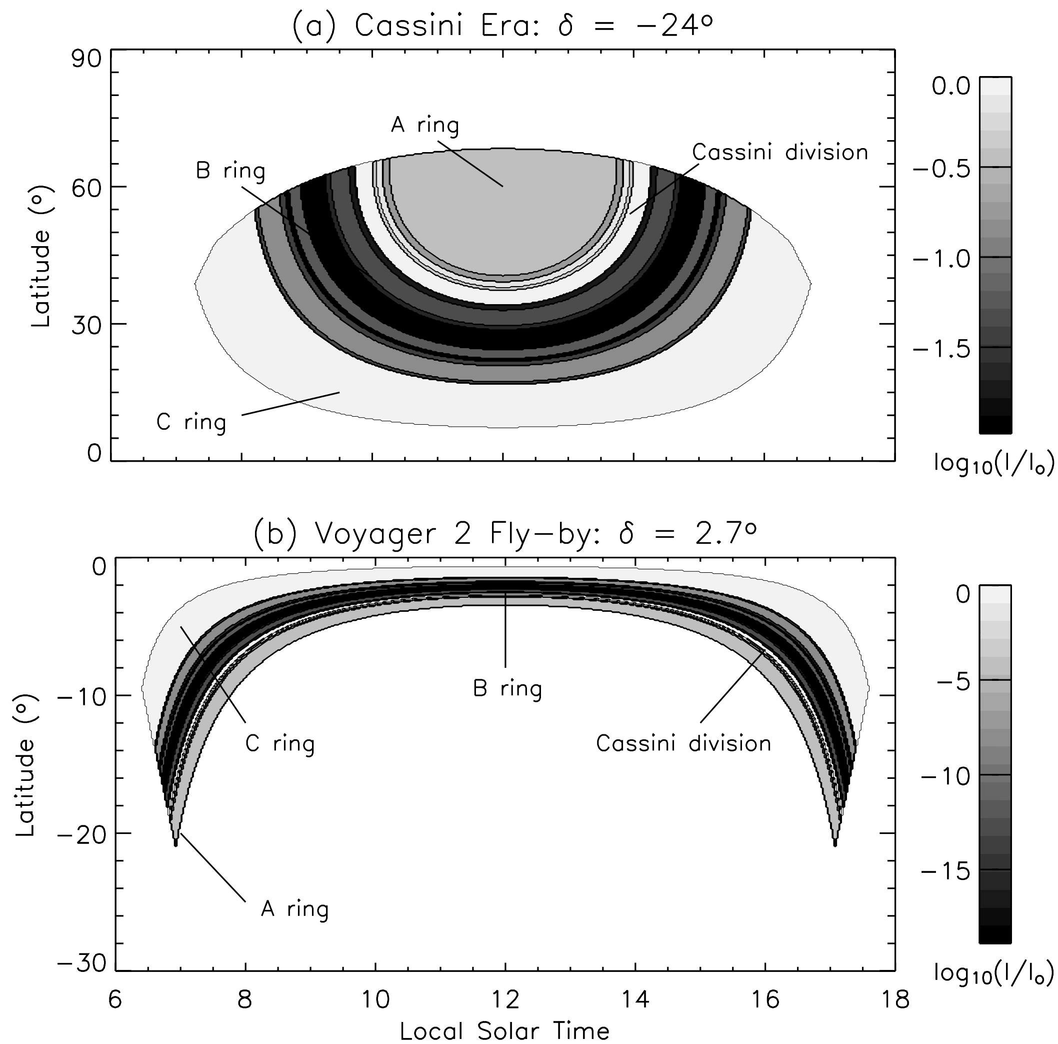

Figure 4